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PERIODIC DECIMAL FRACTIONS

BY W. H. JACKSON

When at school the writer was shown that vulgar fractions with unit numerator and a denominator ending in 9 can be expressed as repeating decimals according to the simple rule illustrated in the following example.

Suppose it is required to express $1/39$ as a repeating decimal. The last figure is 1. To obtain the preceding figures

1×4	$= 4,$	put down 4,
4×4	$= 16,$	put down 6, carry 1,
$6 \times 4 + 1$	$= 25,$	put down 5, carry 2,
$5 \times 4 + 2$	$= 22,$	put down 2, carry 2,
$2 \times 4 + 2$	$= 10,$	put down 0.

The result is given below.

$$\frac{1}{39} = .\dot{0} 2 5 6 4 \dot{1}$$

It is at once clear that the process may be worked in the opposite direction.

$$\begin{aligned} 1 &= 1 + 0 \times 4, \\ 10 &= 2 + 2 \times 4, \\ 22 &= 2 + 5 \times 4, \\ 25 &= 1 + 6 \times 4, \\ 16 &= 4 \times 4, \\ 4 &= 1 \times 4. \end{aligned}$$

Since that time the writer has found no reference to the rule or to the following theorem, from which the rule follows as a special case.

THEOREM. If a be any number prime to 10, and b , less than 10, be chosen so that

$$ab = 10m - 1 \equiv 9, \text{ mod } 10,$$

the number which must be repeated to form the decimal equivalent of $1/a$ may be written

$$b[1 + 10m + (10m)^2 + (10m)^3 + \dots + (10m)^{s-1}] - k10^s,$$

where s is the exponent to which 10 belongs, mod a , and k is a positive integer.

By way of proof it is only necessary to note the identity

$$\frac{-1 + 10^s}{ab} = \frac{-1 + 10^s}{-1 + 10m} = 1 + 10m + (10m)^2 + \dots + (10m)^{s-1} - 10^s \frac{m^s - 1}{ab}$$

and to multiply through by b . When this is done the left-hand side is an integer and we can therefore write, as is otherwise evident,

$$\frac{m^s - 1}{a} = k, \text{ some positive integer.}$$

From this equation it follows that

$$\frac{1}{a} = b \left[\frac{1}{10^s} + \frac{m}{10^{s-1}} + \frac{m^2}{10^{s-2}} + \dots + \frac{m^{s-1}}{10} \right] - k + \frac{1}{a} \cdot \frac{1}{10^s}.$$

When, as in the example above, a ends in a nine, we may choose

$$b = 1, \quad m = \frac{a + 1}{10}.$$

It is easily seen that this formula is equivalent to the rule as stated. The arrival at the end of the process may be recognized by the figures beginning to repeat. Evidently when m is less than 10 this will occur after the application of the rule gives 10 as a product.

The range of numbers for which this method is of practical use is not large, as may be seen from the following table giving values of a corresponding to simple values of m .

<i>m</i>	<i>a</i>	<i>m</i>	<i>a</i>
1	3,9	20	199
2	19	25	249,83
3	29	30	299
4	39,13	40	399,133,57
5	49,7	50	499
6	59	60	599
7	69,23	70	699,233
8	79	80	799
9	89	90	899
10	99,33,11	100	999,333,111
11	109,	200	1999
12	119,17	500	4999

If m has the values 40 or 400 instead of 4, it is necessary to work with two or three figures at a time instead of with one. Thus we find :

$$\frac{1}{399} = .\dot{0}0, 25, 06, 26, 56, 64, 16, 04, 0\dot{1}$$

$$\frac{1}{3999} = .\dot{0}00, 250, 062, 515, 628, 907, 226, ...$$

In this last case the process may be exhibited as below :

$$\begin{aligned}
 1 &= 1 + 000 \times 4 \\
 1000 &= 0 + 250 \times 4 \\
 250 &= 2 + 062 \times 4 \\
 2062 &= 2 + 515 \times 4 \\
 2515 &= 3 + 628 \times 4 \\
 3628 &= 0 + 907 \times 4 \\
 . &
 \end{aligned}$$